Name:_____ Teacher:_____

NZAMT

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School

Level 2 Mathematics and Statistics

Practice Assessment B

91262 (2.7)

Apply calculus methods in solving problems

5 credits

You should answer ALL parts of ALL questions in this booklet.

You should show ALL your working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–10 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO YOUR TEACHER AT THE END OF THE ALLOTTED TIME

You are advised to spend 60 minutes answering the questions in this booklet.

Assessor's use only

QUESTION ONE:

(a) Find the gradient of the function $y = 12 - 3x^2 + 0.3x^3$ when x= -1

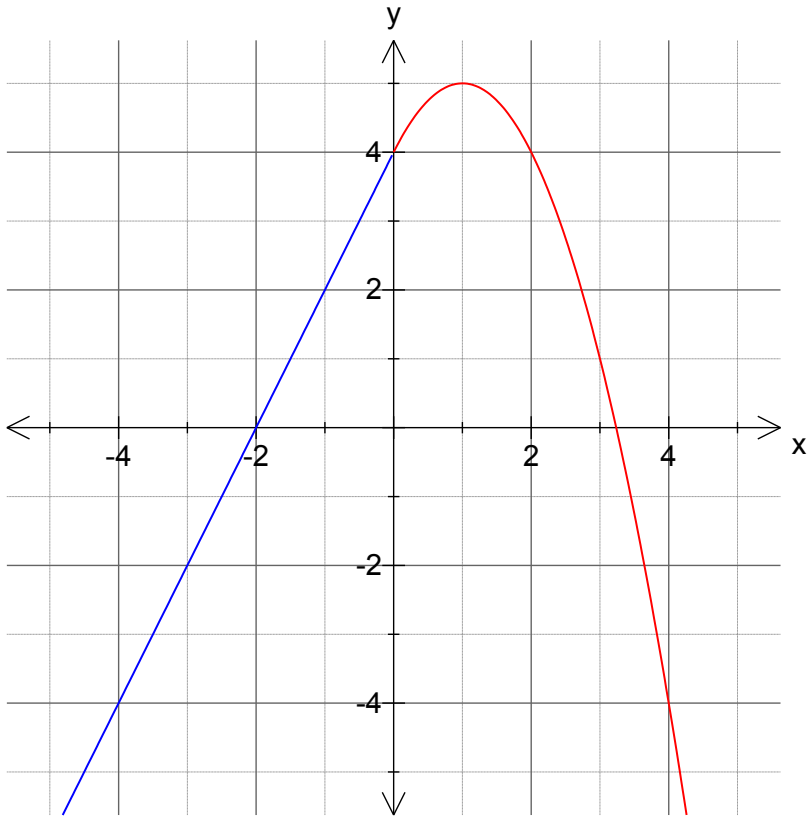
(b) (i) By differentiating $f(x) = 3x^4 - 6x^2 + 8$, determine if the graph is increasing or decreasing when $x = 2$

(ii) Find the coordinates of the turning points on the graph of $f(x) = 3x^4 - 6x^2 + 8$

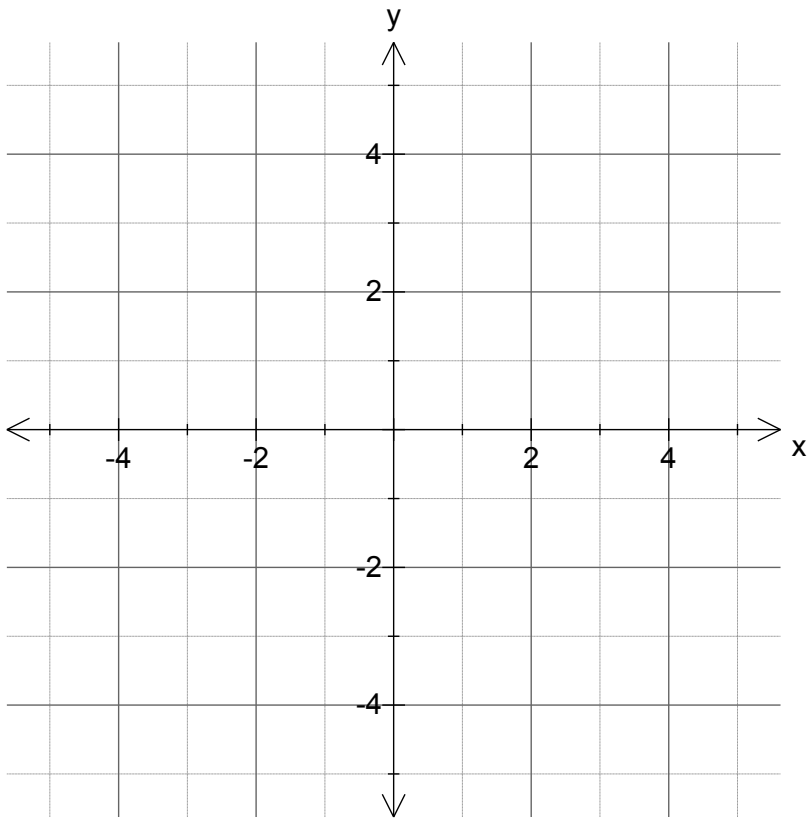
(iii) Give the value of $f''(x)$ for the turning point with the largest x coordinate found in part(ii), and explain what this means.

Assessor's use only

(c) The graph of $y = f(x)$ below is made up of a straight line and a parabola.



On the axes below, sketch the graph of the gradient function.



Assessor's use only

(d) A suitcase in the shape of a cuboid is twice as long as it is high. Certain airlines will allow a total of the length, width and height to be 180cm before making an extra charge. Find the maximum volume a case can have before there is an extra charge and give its dimensions.

Assessor's use only

QUESTION TWO

(a) The gradient function is given by $f'(x) = 6x^2 + 2x - 8$.
What is $f(x)$ if (2,-8) lies on the curve?

(b) Give an equation for the tangent at x = 2 for the function $f(x) = x^5 - 6x^3 + 7$

(c) A remote control car starts moving in a straight line from a fixed point. The velocity in metres per second is given by the formula $v = 3t^2 - 4t - 8$

(i) what is the initial acceleration?

(ii) how far does the car travel in the 3rd second?

(d) An object is moving in a straight line. It has an acceleration of a m sec^{-2} where

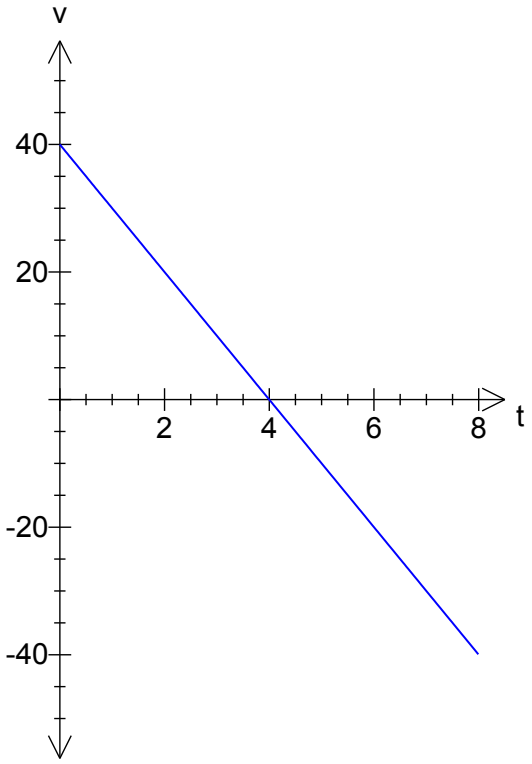
$a = ct + d$

and t is the time since it passed a fixed point on the line.

What is the maximum number of times the object changes direction?
Explain your answer.

Assessor's use only

(e) An object is launched from ground level. The velocity v in metres per second of an object for the first 8 seconds is given by the formula $v = 40 - 10t$ where t is time measured in seconds.
The graph of the velocity is shown below.
The distance is the height above ground level.



Give a full description of the motion of the object after 6 seconds.
You could include in your description: the direction, position, velocity, and acceleration of the object providing values if appropriate.

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Assessor's use only

QUESTION THREE:

(a) The area of a lake in km^2 is given as a function of time t (days) after a storm
$A = 150t - 0.3t^4$

(i) Differentiate A and find the rate at which the area is increasing after 4 days

(ii) When does the lake reach its maximum area and give the maximum area?

(b) For the curve $y = \frac{x^3}{3} - \frac{7x^2}{2} + 6x + 1$

(i) Use calculus methods to find the values of x where the curve is decreasing.

Assessor's use only

(ii) Find the value of x where the curve in part (i) $y = \frac{x^3}{3} - \frac{7x^2}{2} + 6x + 1$ is decreasing the fastest.

(c) A rectangular sheet of metal having a perimeter of 100cm is rolled into a cylinder. What should the dimensions of the metal sheet be for the cylinder to have a maximum volume?
(Formulae: Circumference = $2\pi r$, Area circle = πr^2, Volume of a cylinder = $\pi r^2 h$)

Extra paper for continuing your answers, if required.
Clearly number the question.

Assessor's use only

Question number	

Extra paper for continuing your answers, if required.
Clearly number the question.

Assessor's use only

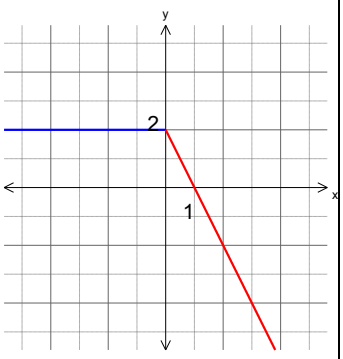
Question number

ASSESSMENT SCHEDULE

Mathematics and Statistics 91262 (2.7): Apply calculus methods in solving problems

Achievement	Merit	Excellence
Apply calculus methods in solving problems involves: • selecting and using methods • demonstrating knowledge of calculus concepts and terms • communicating using appropriate representations.	*Apply calculus methods using relational thinking, in solving problems* must involve one or more of: • selecting and carrying out a logical sequence of steps • connecting different concepts and representations • demonstrating understanding of concepts • forming and using a model • and relating findings to a context, or communicating thinking using appropriate mathematical statements.	*Apply calculus methods using extended abstract thinking, in solving problems* must involve one or more of: • devising a strategy to investigate a situation • demonstrating understanding of abstract concepts • developing a chain of logical reasoning, or proof • forming a generalisation • and using correct mathematical statements, or communicating mathematical insight.

Evidence Statement

One	Expected Coverage	Achievement(u)	Merit(r)	Excellence(t)
NØ = No response; no relevant evidence. N1 = a valid attempt at ONE question. N2 = ONE question demonstrating limited knowledge of calculus techniques.(1u) A3 = TWO of u. A4 = THREE of u. M5 = ONE of r. M6 = TWO of r. E7 = ONE of t, with minor errors ignored. E8 = 2 of t				
(a)	$f(x)' = -6x + 0.9x^2$ $grad = 6.9$	Derivative found and finds the gradient at the point where x = -1.		
(b)(i)	$\frac{dy}{dx} = 12x^3 - 12x$ when x=2, gradient=72 the graph is increasing	differentiates and substitutes in x=2	gradient found and states the curve is increasing	
(b)(ii)	$\frac{dy}{dx} = 12x^3 - 12x = 0$ $x = -1,0,1$ so there is a stationary points at (-1,5),(0,8),(1,5)	equates derivative to 0 and partially solves	finds all three stationary points	
(b)(iii)	y''= $36x^2$-12, at x=1, y''=24 >0 means the point is minimum or that that the gradient is increasing	gives second derivative allow consistency	evaluates y'' for x=1	gives an explanation for positive y''
(c)	Graph of derivative  	derivative drawn, with the derived value of 2 when x<0, and an x-intercept at 1 (the graph does not need to be continuous at x=0)		
(d)	Volume is 0.192m^3.	finds an expression for the volume and derives	equates derivative to 0 and solves	gives the maximum volume

	$180 = x + 2x + y$ $y = 180 - 3x$ Volume $V = 2x^2y$ $V = 2x^2(180 - 3x)$ $= 360x^2 - 6x^3$ $V' = 720x - 18x^2 = 0$ $x = 40$ so max $V = 192000\ cm^3$ or 192 L Dimensions of suitcase are width 40cm, length 80cmandheight 60cm.			

Two	Expected Coverage	u	r	t
NØ = No response; no relevant evidence. N1 = a valid attempt at ONE question. N2 = ONE question demonstrating limited knowledge of calculus techniques.(1u) A3 = TWO of u. A4 = THREE of u. M5 = ONE of r. M6 = TWO of r. E7 = ONE of t, with minor errors ignored. E8 = 2 of t.				
(a)	$y = 2x^3 + x^2 - 8x + c$ $-8 = 2(2)^3 + (2)^2 - 8(2) + c$ $c = -12$ so $y = 2x^3 + x^2 - 8x - 12$	Anti-derivative found, c evaluated		
(b)	$f'(x) = 5x^4 - 18x^2$ point (2,-9), gradient = 8 tangent equation $y = 8x - 25$	derivative found and evaluated at x=2	equation of tangent found	
(c)(i)	$a=6t-4$ when t=0, a=-4	Expression for acceleration and gives the initial acceleration		
(c)(ii)	$s = t^3 - 2t^2 - 8t$ in the third second the car travels $s(3)-s(2)=1$ metre	anti-derives, and finds the distance for t=2 or t=3	finds the distance travelled in the 3 second	
(d)	direction changes a maximum of two times. Change of direction implies v = 0. v is represented by a quadratic function which has a maximum of 2 solutions.	finds the number of time the direction changes	explains the answer.	
(e)	when t=6, $s = 40t - 5t^2$, s=60 m above the ground v=-20ms^{-1} , ,the velocity is 20ms^{-1} and the object is travelling towards the ground a=-10ms^{-2}, the object is accelerating towards the ground	anti-derives to find the distance function and evaluates it when t=6 derives to find the acceleration	describes two of aspects in terms of the objects motion in relation to the ground, numercial quantities for at least one.	describes s, v and a fully by giving values and a description in terms of the relationships 1t=fully correct for 2 aspects 2t = correct for all 3 aspects

Three	Expected Coverage	u	r	t
NØ = No response; no relevant evidence. N1 = a valid attempt at ONE question. N2 = ONE question demonstrating limited knowledge of calculus techniques.(1u) A3 = TWO of u. A4 = THREE of u. M5 = ONE of r. M6 = TWO of r. E7 = ONE of t, with minor errors ignored E8 = 2 of t				
(a) (i)	$\frac{dA}{dt} = 150 - 1.2t^3$ when t=4, rate = 73.2 km^2/day	differentiates and finds the rate		
(a) (ii)	$\frac{dA}{dt} = 150 - 1.2t^3 = 0$ t=-5 or 5, after 5 days, A=562.5km^2	equates the derivative to 0 and finds t=5	derives and finds the maximum Area	
(b) (i)	$\frac{dy}{dx} = x^2 - 7x + 6 \leq 0$ decreasing between 1 and 6	derivative and an equation or inequation formed	gives interval for x where the function is decreasing	
(b) (ii)	decreasing the fastest where $\frac{dy}{dx} = x^2 - 7x + 6$ reaches a min $y'' = 2x - 7 = 0$ $x = 3.5$	derives to get y''	gives y'' and equates to zero	derives and finds the x value where the gradient function is minimum
(c)	$100 = 2x + 2y$ so $y = 50 - x$ $V = \pi r^2 x$, since $y = 2\pi r$ then $r = \frac{y}{2\pi}$		forms the equation for V, derives	forms the equation, derives, and solves the problem, both dimensions given.